Name:____________________________ Date:___________________________

Math 10/11 Honors: Section 3.4 Recursive Sequences and Series

1. Given each recursive sequence, find the indicated term:

a) $t_1 = 3,\ \ t_{n+1} = 2\times t_n + 3,\ t_8 =$	b) $t_1 = 2,\ t_2 = 3,\ \ t_{n+2} = t_n + 3t_{n+1},\ t_6 =$
c) If $a_1 = 3$ and $a_{n+1} = 2a_n - 1$, what is a a_{10}?	d) If $a_{10} = 10$ and $a_{n+1} = \dfrac{a_n - 1}{2} + 4$, what is a a_7?
e) If $a_1 = 1$, $a_2 = 2$, and $a_n = 3a_{n-2} + a_{n-1}$ evaluate a_6	f) If $a_{10} = 10$ & $a_n = \dfrac{a_{n-1}}{2} + 4$, what is the value of a_7 ?

2. Find the sum of the following series:

a) $3 + 12 + 27 + 48 + 75.......3000$	b) $1^2 + 3^2 + 5^2 + ... + 1001^2$

c) $\frac{1}{1(2)}+\frac{1}{2(3)}+\frac{1}{3(4)}+.......+\frac{1}{19(20)}$	d) $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)......\left(1-\frac{1}{n}\right)=\frac{1}{45}$ $n=?$

3. The first term of a sequence is 2007. Each term, starting with the second, is the sum of the cubes of the digits of the previous term. What is the 2007th term?

4. What is the sixth tem of a recursively defined sequence with its first term defined as $a_1=8$ and all subsequent terms defined as $a_n=\left(8-a_{n-1}\right)^2+7$?

5. Each term in the serious below has the form $\frac{1}{(n)(n+3)}$. Find the sum of this series:

$$\frac{1}{(1)(4)}+\frac{1}{(4)(7)}+\frac{1}{(7)(10)}+...\frac{1}{(298)(301)}$$

6. A school has a phone tree that begins with the principle being responsible for calling four teachers. Each teacher afterwards is responsible for calling two other teachers. If each call takes exactly 2 minutes and the first call started at 9:00am, how long will it take to contact all the teachers if there are 250 teachers in the school?

7. A sequence of numbers $t_1, t_2, t_{3...}$ is defined by: $t_1 = 7$ and $t_{n+1} = \sqrt{|(t_n)^2 - 16|}$. What is the value of t_{80} ?

8. A function $f(x)$ has the following three properties. Calculate $f(6)$

   i) $f(1) = 1$,   ii) $f(2x) = 4f(x) + 6$ iii) $f(x+2) = f(x) + 12x + 12$

9. The sequence of numbers $t_1,\ t_2,\ t_3,\ ....$ is defined by $t_1 = 2$ and $t_{n+1} = \dfrac{t_n - 1}{t_n + 1}$, for every positive integer "n".

   Determine the numerical value of t_{999}

10. In seven term sequence, 5, p , q , 13, r , 40, x , each term after the third term is the sum of the preceding three terms. The value of x is

    a) 21   b) 61   c) 67   d) 74   e) 80

11. The sequence 2, 5, 10, 50, 500,….. is formed so that each term after the second is the product of the two previous terms. The 15th term ends with exactly "k" zeroes. What is the value of "k"?

12. The function $f(x)$ has the property that $f(x+y)=f(x)+f(y)+2xy$, for all positive integers x and y. If $f(1)=4$, then the numerical value of $f(8)$ is
a) 72 b) 84 c) 88 d) 64 e) 80

13. A sequence t_1, t_2, …, t_n, … is defined as follows: i) $t_1=14$ ii) $t_k=24-5t_{k-1}$ for each $k\geq 2$. For every positive integer n, t_n can be expressed as $t_n = p\cdot q^n + r$, where p, q and r are constants. What is the value of $p+q+r$?
a) -5 b) -3 c) 3 d) 17 e) 31

14. In a sequence, every term after the second term is twice the sum of the two preceding terms. The seventh term of the sequence is 8, and the ninth term is 24. What is the eleventh term of the sequence?
a) 160 b) 304 c) 28 d) 56 e) 64

15. If $a_1=\dfrac{1}{1-x}$, $a_2=\dfrac{1}{1-a_1}$, and $a_n=\dfrac{1}{1-a_{n-1}}$, for $n\geq 2$, $x\neq 1$ and $x\neq 0$, then a_{107} is
a) $\dfrac{1}{1-x}$ b) x c) $-x$ d) $\dfrac{x-1}{x}$ e) $\dfrac{1}{x}$

16. The first term in a sequence of numbers is $t_1 = 5$. Successive terms are defined by the statement $t_n - t_{n-1} = 2n+3$ for $n \geq 2$. What is the value of t_{50}?

a) 2700 b) 2702 c) 2698 d) 2704 e)2706

17. The Fibonacci sequence begins: 1, 1, 2, 3, 5, 8, 13, 21… (Each number beyond the second number is the sum of the previous two numbers.) The notation f_n means the n^{th} number, for example $f_4 = 3$ and $f_7 = 13$.

a) Which of the following terms in the Fibonacci sequence are odd? Explain your conclusions.

f_{38}, f_{51}, f_{150}, f_{200}, f_{300}

b) Which of the following terms in the Fibonacci sequence are divisible by 3? Explain your conclusions.

f_{48}, f_{75}, f_{196}, f_{379}, f_{1000}

23. Let t_n equal the integer closest to $\sqrt{n}$.
For example, $t_1 = t_2 = 1$ since $\sqrt{1} = 1$ and $\sqrt{2} \approx 1.41$ and $t_3 = 2$ since $\sqrt{3} \approx 1.73$.
The sum $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} + \cdots + \frac{1}{t_{2008}} + \frac{1}{t_{2009}} + \frac{1}{t_{2010}}$ equals

(A) $88\frac{1}{6}$ **(B)** $88\frac{1}{2}$ **(C)** $88\frac{2}{3}$ **(D)** $88\frac{1}{3}$ **(E)** 90

